

# High School Content Expectations Companion Document



# MATHEMATICS

## Pre- and Post- MME Expectations

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## **Pre- and Post- MME Expectations**

### **Rationale for Identifying**

The Michigan Merit Curriculum requires that students earn 4 credits of mathematics. The required mathematics High School Content Expectations (HSCE) represent 3 credits. The 4<sup>th</sup> credit is at the discretion of the district. The mathematics portion of the Michigan Merit Exam (MME) assesses student's understanding of the required mathematics HSCE, at the standard level. The MME is based in large part on the ACT with a Michigan portion testing those standards not addressed by the ACT. Since the MME is administered in March of a student's junior year, it is reasonable to expect that not all of the required HSCE will be covered by that time. Therefore, with input from ACT, these expectations could be taught after the MME, with very little effect on students' preparation for the MME. Rationales are included under each topic identified as post-MME so that users have an understanding of the potential impact of delaying the teaching of these topics.

### **Suggestions for Use**

- In regards to personal curriculum modifications, these can be a tool to decide what content they want to teach in the .5 credit of Algebra II or the 1<sup>st</sup> credit of a 2-credit Algebra II course.
- For districts developing a 1-credit Algebra II course over 2 years, this would be a tool to help design each year of the course.
- Similarly, this would provide guidance to districts implementing an integrated sequence of instruction for high school mathematics.
- Teachers could use this document in planning the pace of their instruction for a 1 year Algebra II course.
- Guidance for designing the Michigan portion of the mathematics MME.

## **Pre-MME Mathematics Expectations**

**All of the expectations found in Algebra I and Geometry should be taught before the MME, whether they are taught in a traditional or integrated sequence. The following Algebra II expectations should also be taught before the MME (30 expectations):**

### **L1.2 Representations and Relationships**

- L1.2.1 Use mathematical symbols to represent quantitative relationships and situations.

### **L1.3 Counting and Probabilistic Reasoning**

- L1.3.1 Describe, explain, and apply various counting techniques; relate combinations to Pascal's triangle; know when to use each technique.
- L1.3.2 Define and interpret commonly used expressions of probability.
- L1.3.3 Recognize and explain common probability misconceptions such as "hot streaks" and "being due."

### **L2.1 Calculation Using Real and Complex Numbers**

- L2.1.3 Explain the exponential relationship between a number and its base 10 logarithm, and use it to relate rules of logarithms to those of exponents in expressions involving numbers.
- L2.1.5 Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.

### **L2.3 Measurement Units, Calculations, and Scales**

- L2.3.2 Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements; solve applied problems.

### **A1.1 Construction, Interpretation, and Manipulation of Expressions**

- A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.
- A1.1.4 Add, subtract, multiply, and simplify polynomials and rational expressions.
- A1.1.5 Divide a polynomial by a monomial.
- A1.1.6 Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms, including the inverse relationship between exponents and logarithms.

### **A1.2 Solutions of Equations and Inequalities**

- A1.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.
- A1.2.5 Solve polynomial equations and equations involving rational expressions and justify steps in the solution.

- A1.2.7 Solve exponential and logarithmic equations and justify steps in the solution.
- A1.2.8 Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.
- A1.2.9 Know common formulas and apply appropriately in contextual situations.

### **A3.2 Exponential and Logarithmic Functions**

- A3.2.2 Interpret the symbolic forms and recognize the graphs of logarithmic functions.
- A3.2.3 Apply properties of exponential and logarithmic functions.

### **G1.7 Conic Sections and Their Properties**

- G1.7.1 Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.
- G1.7.2 Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.
- G1.7.3 Graph ellipses and hyperbolas with axes parallel to the x- and y-axes, given equations.

### **S1.1 Producing and Interpreting Plots**

- S1.1.1 Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.
- S1.1.2 Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.

### **S1.2 Measures of Center and Variation**

- S1.2.1 Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.
- S1.2.2 Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.
- S1.2.3 Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

#### **S4.1 Probability**

- S4.1.1 Understand and construct sample spaces in simple situations.
- S4.1.2 Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.

#### **S4.2 Application and Representation**

- S4.2.1 Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.
- S4.2.2 Apply probability concepts to practical situations in such settings as finance, health, ecology, or epidemiology to make informed decisions.

**[Expectations from Standard A2 appear on the MME in the context of logarithmic functions (Algebra II) in addition to all of the Algebra I function families.]**

#### **A2.1 Definitions, Representations, and Attributes of Functions**

- A2.1.1 Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.
- A2.1.2 Read, interpret, and use function notation, and evaluate a function at a value in its domain.
- A2.1.3 Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.
- A2.1.6 Identify the zeros of a function, the intervals where the values of a function are positive or negative, and describe the behavior of a function as  $x$  approaches positive or negative infinity, given the symbolic and graphical representations.
- A2.1.7 Identify and interpret the key features of a function from its graph or its formula(s).

#### **A2.2 Operations and Transformations**

- A2.2.1 Combine functions by addition, subtraction, multiplication, and division.
- A2.2.2 Apply given transformations to basic functions, and represent symbolically.
- A2.2.3 Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.

#### **A2.3 Families of Functions**

- A2.3.1 Identify a function as a member of a family of functions based on its symbolic or graphical representation; recognize that different families of functions have different asymptotic behavior.
- A2.3.3 Write the general symbolic forms that characterize each family of functions.

#### **A2.4 Models of Real-World Situations Using Families of Functions**

- A2.4.1 Identify the family of function best suited for modeling a given real-world situation.

## Post-MME Mathematics Expectations

The following Algebra II expectations could be taught post MME (21 expectations):

### L2.2 Sequences and Iteration

- L2.2.1 Find the  $n$ th term in arithmetic, geometric, or other simple sequences.
- L2.2.2 Compute sums of finite arithmetic and geometric sequences.
- L2.2.3 Use iterative processes in such examples as computing compound interest or applying approximation procedures.

*[This topic is listed in the Intermediate Algebra portion of the ACT, which according to ACT,<sup>i</sup> is about 15% of the test. It is also listed in the 33-36 score range of the College Readiness Standards for Mathematics<sup>ii</sup>]*

### L2.4 Understanding Error

- L2.4.1 Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.
- L2.4.2 Describe and explain round-off error, rounding, and truncating.
- L2.4.3 Know the meaning of and interpret statistical significance, margin of error, and confidence level.

*[Correspondence with a Senior Test Development Associate in Math at ACT indicates that while it is possible that some of these expectations could be tested it is not likely]*

### A3.6 Rational Functions

- A3.6.1 Write the symbolic form and sketch the graph of simple rational functions.
- A3.6.2 Analyze graphs of simple rational functions and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.

*[While not mentioned specifically, could appear in the Intermediate Algebra section of the ACT which is about 15% of the test]*

### A3.7 Trigonometric Functions

- A3.7.1 Use the unit circle to define sine and cosine; approximate values of sine and cosine; use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.
- A3.7.2 Use the relationship between degree and radian measures to solve problems.
- A3.7.3 Use the unit circle to determine the exact values of sine and cosine, for integer multiples of  $\pi/6$  and  $\pi/4$ .

- A3.7.4 Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.
- A3.7.5 Graph transformations of basic trigonometric functions (involving changes in period, amplitude, phase, and midline) and understand the relationship between constants in the formula and the transformed graph.

## **A1.2 Solutions of Equations and Inequalities**

- A1.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals.

*[Trigonometry is about 7% of the ACT;<sup>i</sup> unit circle trigonometry appears in the 33-36 score range of the College Readiness Standards for Mathematics<sup>ii</sup>]*

## **S1.3 The Normal Distribution**

- S1.3.1 Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.
- S1.3.2 Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.
- S1.3.3 Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.
- S1.3.4 Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.

*[Correspondence with a Senior Test Development Associate in Math at ACT indicates that while it is possible that some of these expectations could be tested it is not likely]*

## **S3.1 Data Collection and Analysis**

- S3.1.1 Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.
- S3.1.2 Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.
- S3.1.3 Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

*[Correspondence with a Senior Test Development Associate in Math at ACT indicates that while it is possible that some of these expectations could be tested it is not likely]*



[Expectations from Standard A2 are *not* likely to appear on the MME in the context of *trigonometric* and *rational* functions.]

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- A2.2.3 Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.

## **A2.3 Representations of Functions**

- A2.3.1 Identify a function as a member of a family of functions based on its symbolic or graphical representation; recognize that different families of functions have different asymptotic behavior.
- A2.3.3 Write the general symbolic forms that characterize each family of functions. (e.g.,  $f(x) = A_0a^x$ ;  $f(x) = A\sin Bx$ )

## **A2.4 Models of Real-World Situations Using Families of Functions**

- A2.4.1 Identify the family of function best suited for modeling a given real-world situation.
- A2.4.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers.
- A2.4.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled.

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<sup>i</sup> ACT, Inc. (2005). *Your Guide to the ACT*. pg. 8. Retrieved from the ACT website: <http://www.act.org/aap/pdf/YourGuidetoACT.pdf>.

<sup>ii</sup> ACT, Inc. (2005). pg. 9.